BBC RD 1977/1

1 9 7 4
THE QUEEN'S AWARD
TO INDUSTRY

RESEARCH DEPARTMENT REPORT

A 4-PHASE DIFFERENTIAL-PHASE-SHIFT-KEYING STEREO SOUND SYSTEM:
carrier recovery methods

W.A. Mahadeva, B.A.

Research Department, Engineering Division
THE BRITISH BROADCASTING CORPORATION January 1977

BBC RD 1977/1
UDC 621.376.4

A 4-PHASE DIFFERENTIAL-PHASE-SHIFT-KEYING STEREO SOUND SYSTEM: CARRIER RECOVERY METHODS

W.A. Mahadeva, B.A.

Summary

A system for carrying high-quality stereophonic digital sound signals using 4-phase differential-phase-shift-keying modulation is being developed by the BBC. The system could find applications in high-quality sound links or as a means of transmitting the sound signal for television broadcasting from satellites.

Recovering the carrier at the receiver for synchronous demodulation was one of the problems encountered when developing the system. This report compares three methods of carrier recovery, namely phase-multiplication, remodulation, and baseband remodulation, and describes the measured performances of prototype equipment using the first and last of these three methods.

Baseband remodulation is the method recommended because it offers good performance and is the cheapest of the three methods to implement.

Issued under the authority of

G D Monteath

Head of Research Department

Research Department, Engineering Division,
BRITISH BROADCASTING CORPORATION

January 1977

(EL-124)

© BBC 2004. All rights reserved. Except as provided below, no part of this document may be reproduced in any material form (including photocopying or storing it in any medium by electronic means) without the prior written permission of BBC Research & Development except in accordance with the provisions of the (UK) Copyright, Designs and Patents Act 1988.

The BBC grants permission to individuals and organisations to make copies of the entire document (including this copyright notice) for their own internal use. No copies of this document may be published, distributed or made available to third parties whether by paper, electronic or other means without the BBC's prior written permission. Where necessary, third parties should be directed to the relevant page on BBC's website at http://www.bbc.co.uk/rd/pubs/ for a copy of this document.

A 4-PHASE DIFFERENTIAL-PHASE-SHIFT-KEYING STEREO SOUND SYSTEM: CARRIER RECOVERY METHODS

Section Title Page

Summary .... Title Page

1. Introduction 1

2. The 4-phase d.p.s.k. system 1

2.1. Modulation 1
2.2. Demodulation 1

3. Carrier recovery 2

3.1. General 2
3.2. Carrier recovery by phase-multiplication 2
3.3. Carrier recovery by remodulation 2
3.4. Carrier recovery by baseband remodulation 4
3.5. Effects of restricted bandwidth 4

4. Comparison of carrier recovery methods 6

4.1. General 6
4.2. Sensitivity to instrumental phase drifts 6
4.3. Dependence upon modulating sequence 6
4.4. Sensitivity to level of input signal 6
4.5. The phase-error function 6
4.6. Simplicity of instrumentation 7

5. Theoretical and measured performance of prototype units 7

6. Practical aspects of baseband remodulation carrier recovery unit design 8

7. Conclusions 10

8. References 10

9. APPENDIX I: Equivalence of the conventional remodulation method and the baseband remodulation method of carrier recovery 11

10. APPENDIX II: Baseband remodulation carrier recovery unit: Analysis and design 13

10.1. Analysis 13
10.2. Performance of a linearised model 13
10.3. Design objectives 14
10.4. The design of the experimental circuits 15
    10.4.1. Basic circuit parameters 15
    10.4.2. Static phase accuracy 15
    10.4.3. Phase jitter due to noise 15

Section Title Page

10.4.4. Loop damping factor and natural frequency 15
10.4.5. Pull-in range and hold-in range 15
10.4.6. The effect on the p.l.l. of modulation on the d.p.s.k. signal 15
10.5. The experimental loop filter 15
10.6. Performance of the experimental circuit 16

A 4-PHASE DIFFERENTIAL-PHASE-SHIFT-KEYING STEREO SOUND SYSTEM: CARRIER RECOVERY METHODS

W.A. Mahadeva, B.A.

1. Introduction

Increasing use is being made of digital techniques to carry high-quality sound signals because such coding of an analogue signal greatly increases the immunity of the signal to transmission impairments.

At present, high-quality sound signals are distributed in digital form to most of the main BBC transmitters; for this purpose thirteen individual audio channels are multiplexed together to form a composite p.c.m. bit-stream with a bit-rate of 6·336 Mbit/s. Another application of digital sound transmission is found in parts of the television network; in this case the sound information is carried in digital form during the line-synchronising pulses of the video signals.

One area where digital techniques could also be applied with advantage is in contribution links and, in particular, outside broadcast (OB) radio links. At present, the audio signal is carried by a frequency-modulated carrier. The range of the equipment is found to be acceptable for monophonic signals but its range is greatly reduced when transmitting stereophonic signals. If a digital system were used, it might be possible to achieve stereo operation over a much greater range.

Primarily with this application in mind, an investigation has been carried out into a system for transmitting signals that are digitally-coded at a rate appropriate for two high-quality sound signals. It is hoped that the knowledge gained from this work will be helpful when considering future applications of digital techniques to sound-signal distribution and, possibly, in direct-broadcasting from satellites.

The modulation system used in the experimental equipment was 4-phase differential-phase-shift-keying (d.p.s.k.), in which the digital signal is carried as a series of carrier phase-changes between any of four rest-states. The advantage of differential coding is that, since only *changes* in carrier phase need to be detected, there is no need for the receiver to identify any particular one of the four rest-states and hence there is no need to radiate a reference carrier-phase.

This report describes and compares three methods of d.p.s.k. carrier recovery, and describes the measured performance of prototype equipment using two of the methods.

2. The 4-phase d.p.s.k. system

2.1. Modulation

The 4-phase d.p.s.k. system was chosen because it offers a good compromise between the conflicting requirements of low carrier power (for a given range), narrow bandwidth, and low cost.[1] In this system, information is transmitted by changing the phase of a carrier in steps which are integral multiples of 90°. The phase of the carrier as transmitted has four rest-states shown as 1, 2, 3 and 4 in Fig. 1. The phase alters from one state to another, according to the modulation information, at a fixed rate called the 'symbol rate'. Since the phase of the carrier can change by any one of four possible values, each phase-change carries, in binary terms, two bits of information. Thus the bit-rate of the system is twice the symbol rate. The carrier frequency chosen for the prototype equipment described in this report was 10·7 MHz. The conclusions of this report are also valid for other carrier frequencies provided that they are greater than about three times the bit-rate. The symbol rate was 326 ksymbols/s, corresponding to a bit-rate of 652 kbit/s. This was thought to be adequate for two 10-bit companded p.c.m. sound signals, each with 15 kHz bandwidth.

2.2. Demodulation

At the receiver, the signal can be demodulated in either of two ways. One is to measure phase-changes directly by comparing the phases of the signal from one symbol to the next. The other is to employ synchronous demodulation by which the phase of the incoming signal is compared with that of a locally-derived reference carrier. The former, non-synchronous, method has a much poorer noise performance[1] and requires a delay of

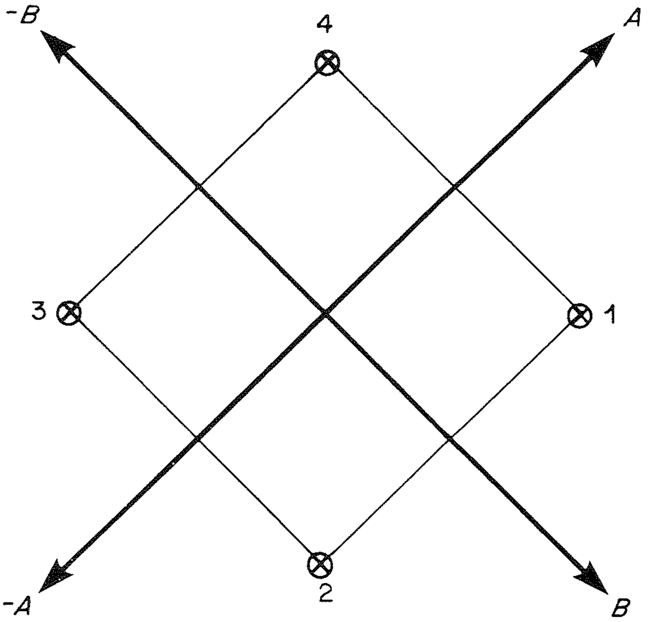

Fig. 1 - Vector diagram for 4-phase d.p.s.k. system

one symbol, accurate to within a very small fraction of one cycle at the carrier frequency. Synchronous demodulation is therefore preferable and is the only system dealt with in this report.

The signal can be synchronously demodulated by using two carriers in phase quadrature. In Fig. 1, the four possible rest-states for the incoming signal are shown as 1, 2, 3 and 4. If at the receiver, we have a reference carrier A and a quadrature carrier B, we can, by synchronous demodulation using these two carriers, identify the phase of the incoming signal. If, for instance, the carrier is in state 1, both demodulators would give a positive output. In state 2, the synchronous demodulator using carrier A would give a negative output whilst that using carrier B would give a positive output. States 3 and 4 can also be recognised in a similar manner. The phase-changes of the incoming carrier which occur from one symbol to the next can be deduced from the succession of positive or negative outputs from the two demodulators. Since information is transmitted by means of changes of carrier phase it does not matter which of the four phases A, B, $-A$, or $-B$ is chosen for the reference carrier but its phase must, of course, remain stable. The problem of carrier recovery is to produce a reference carrier that remains stable in one of the four phases A, B, $-A$ or $-B$. Note that these phases are preferably set at $\pi/4$ from those of the incoming signal.

3. Carrier recovery

3.1. General

The amplitude of the carrier-frequency component of a 4-phase d.p.s.k. signal depends upon the modulation sequence and, for certain sequences, it may be very small, or even zero. A non-linear operation must therefore be performed on the signal to generate a component at carrier frequency (or some multiple of it). This component can then be selected using a tuned circuit or phase-locked loop. The following three methods were considered for the work described in this report:

(i) Phase-multiplication
(ii) Remodulation
(iii) Baseband remodulation

Methods (i) and (ii) have previously been described.[1,2,3,4]

3.2. Carrier recovery by phase-multiplication

If a fourth-order non-linear operation is performed on the d.p.s.k. signal, a component at four-times carrier frequency is generated. This is because if the phase of the input signal changes instantaneously by an integral multiple of 90°, the phase of the four-times carrier-frequency component changes instantaneously by the same multiple of 360°. That is, no phase-change takes place. Since the phase-states of a d.p.s.k. carrier (1, 2, 3 and 4 in Fig. 1) are separated by integral multiples of 90° from each other the phase of the resulting four-times carrier-frequency component is constant. A voltage-controlled oscillator (VCO) can therefore be phase-locked to this component, and its output frequency then divided by four to provide a recovered carrier. Fig. 2 is a block diagram of the instrumentation of this process, which we can call 'phase-multiplication'.

3.3. Carrier recovery by remodulation

This method of carrier recovery generates a component at the carrier frequency by cancelling the modulation on the d.p.s.k. signal. With reference to Figs. 3(a) and 3(b), assume the voltage controlled oscillator is running at the carrier frequency but with a phase error of θ

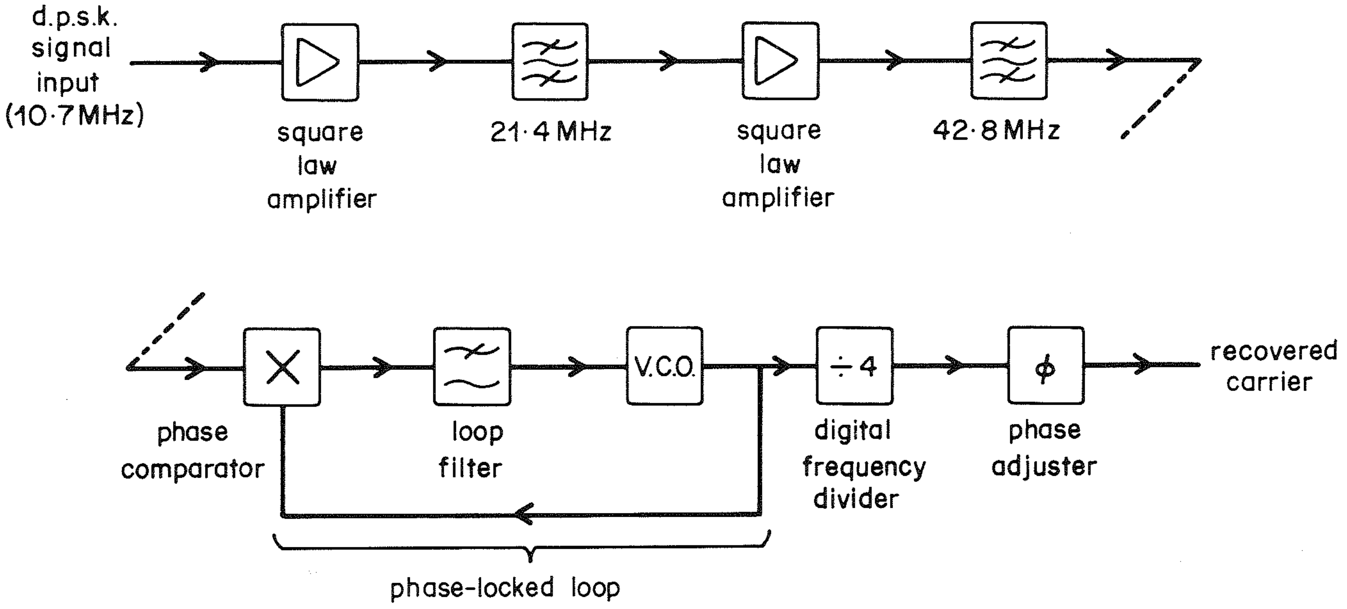

Fig. 2 - Carrier recovery by phase-multiplication

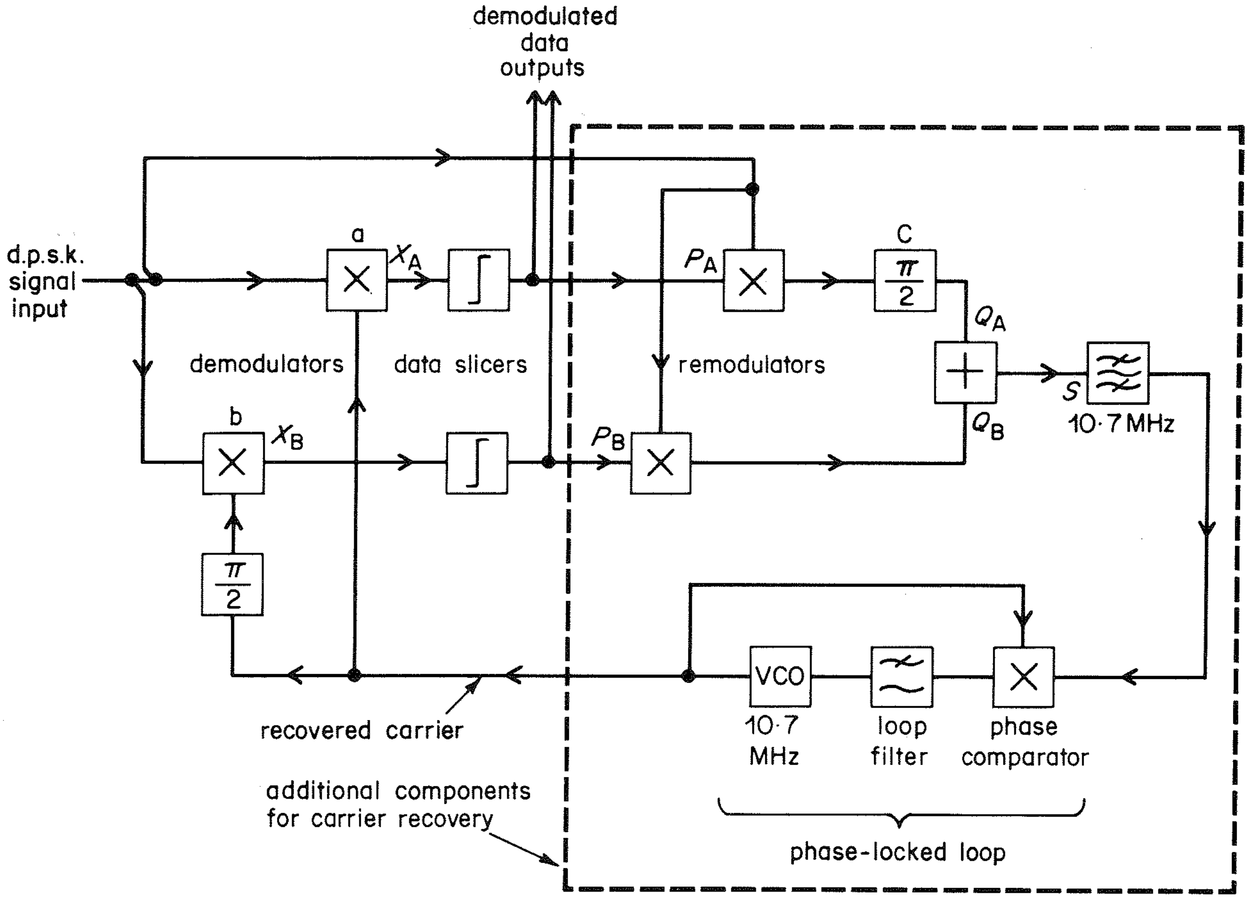

Fig. 3(a) - Carrier recovery by remodulation

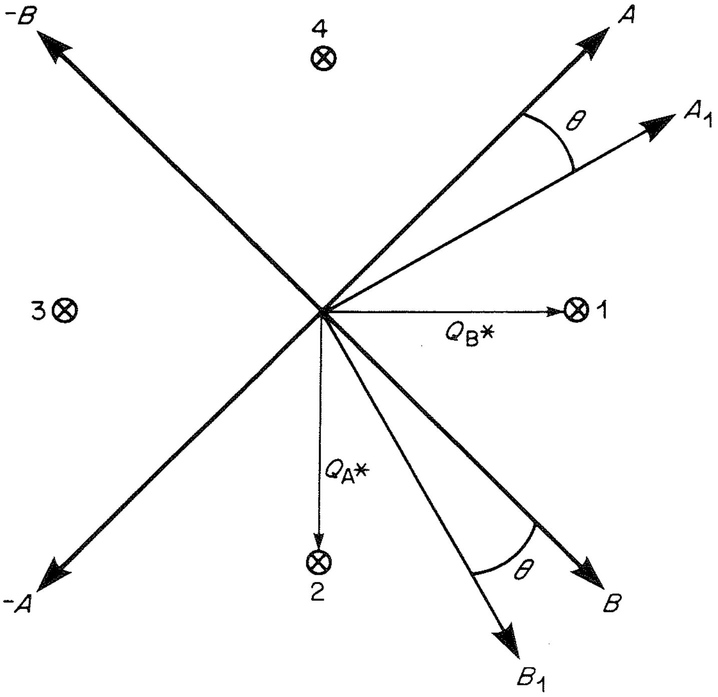

Fig. 3(b) - Vector diagram for carrier recovery by remodulation

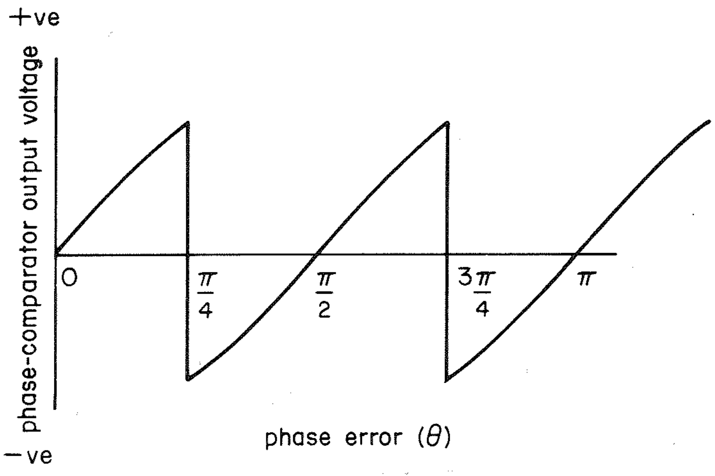

Fig. 3(c) - Phase-error function for carrier recovery by remodulation

($|\theta| < 45°$) with respect to the phase of A (i.e. with phase of A_1). The demodulators a and b will be operating with carriers A_1 and B_1 respectively. Even with the phase error θ, it is possible to distinguish between the four rest-states of the d.p.s.k. signals. Assume for the moment that the d.p.s.k. signal is constant in any one of its four rest-states. The demodulator outputs X_A and X_B will be d.c. voltages whose polarity will be determined by the particular rest-state of the d.p.s.k. signal. The data slicers, which can be regarded as amplitude limiters, give the outputs P_A and P_B that will each be +1 or −1 depending on the phase state of the d.p.s.k. signal.* If either of the output signals P_A or P_B is +1 the remodulator fed with this signal passes, without change, the d.p.s.k. signal which is fed into its other port. If the signal P_A or P_B is −1 the d.p.s.k. signal is phase-reversed by the corresponding remodulator. This remodulation process, and the $\pi/2$ phase shifter C, are so arranged that, regardless of which of the four rest-states the d.p.s.k. carrier is in, the phase of the signal S (Fig. 3(a)) has the phase of B in Fig. 3(b). The phase-locked loop, locked to the signal S, will therefore drive the phase of its voltage-controlled oscillator towards the phase of A.

This description applies when the d.p.s.k. signal is at rest in one of its four rest-states. The situation is not changed however if the d.p.s.k. signal is modulated such that its phase changes instantaneously between the four rest-states at the symbol rate. We have assumed that $|\theta| < 45°$. If $|\theta| > 45°$ the phase of the voltage-controlled oscillator will, by arguments of symmetry, be driven toward one of the remaining three vectors B, $-A$ or $-B$.

The acquisition mechanism of the remodulation unit is similar to that of normal phase-locked loops. The output voltage from the phase comparator of the phase-locked loop varies as a function of the phase error (θ) as shown in Fig. 3(c). If the voltage-controlled oscillator starts at an incorrect frequency, lock will be acquired in exactly the manner of a conventional phase-locked loop, with this phase-error function.

3.4. Carrier recovery by baseband remodulation

As shown in Appendix I, this method of carrier recovery is the baseband equivalent of the remodulation method of carrier recovery described above. To understand and analyse its operation, however, it is better to deduce its mode of operation from first principles. Fig. 4(a) is a block diagram illustrating the instrumentation of the system; the phase relationships of the signals involved are shown in Fig. 4(b). Assume that the voltage-controlled oscillator is operating at the correct frequency but with a phase error θ ($|\theta| < 45°$). In Fig. 4(b), if the reference and quadrature carriers (A_1 and B_1) are in the correct phases (i.e. if $\theta = 0$), the moduli of the d.c. outputs X_A and X_B from the two demodulators will be equal when the d.p.s.k. signal is at rest in any one of its four rest-states. Any error (θ) in the carrier phase will make the moduli of the d.c. voltages X_A and X_B unequal. (In Fig. 4(*b*) the values of X_A and X_B with a recovered carrier phase error θ are denoted X'_A and X'_B). The baseband remodulation method uses this property to produce an error voltage E, (see Fig. 4(a)) the magnitude and polarity of which bears a fixed relationship to the phase error of the carrier, regardless of which of the four rest-states the signal occupies (see Appendix II for a fuller description). The data slicers and remodulators operate as in the remodulation method (see Section 3.3. above) except that the remodulators operate on and produce d.c. signals. This description has assumed that the phase of the d.p.s.k. signal is constant in one of its four rest-states, but it applies equally if the phase of the signal is switched instantaneously between any of the four rest-states. The error voltage produced by the unit bears the same relationship to the phase error as that shown in Fig. 3(c) which is produced by the conventional remodulation method. The baseband remodulation method thus acquires lock, and eliminates phase error, in the same manner as the conventional remodulation method.

3.5. Effects of restricted bandwidth

In a d.p.s.k. system with a finite bandwidth the signal takes some time to make transitions between the states. The descriptions given above of the various methods of carrier recovery assumed instantaneous trans-

---

* It is assumed for convenience that the data slicers give outputs of ±1.

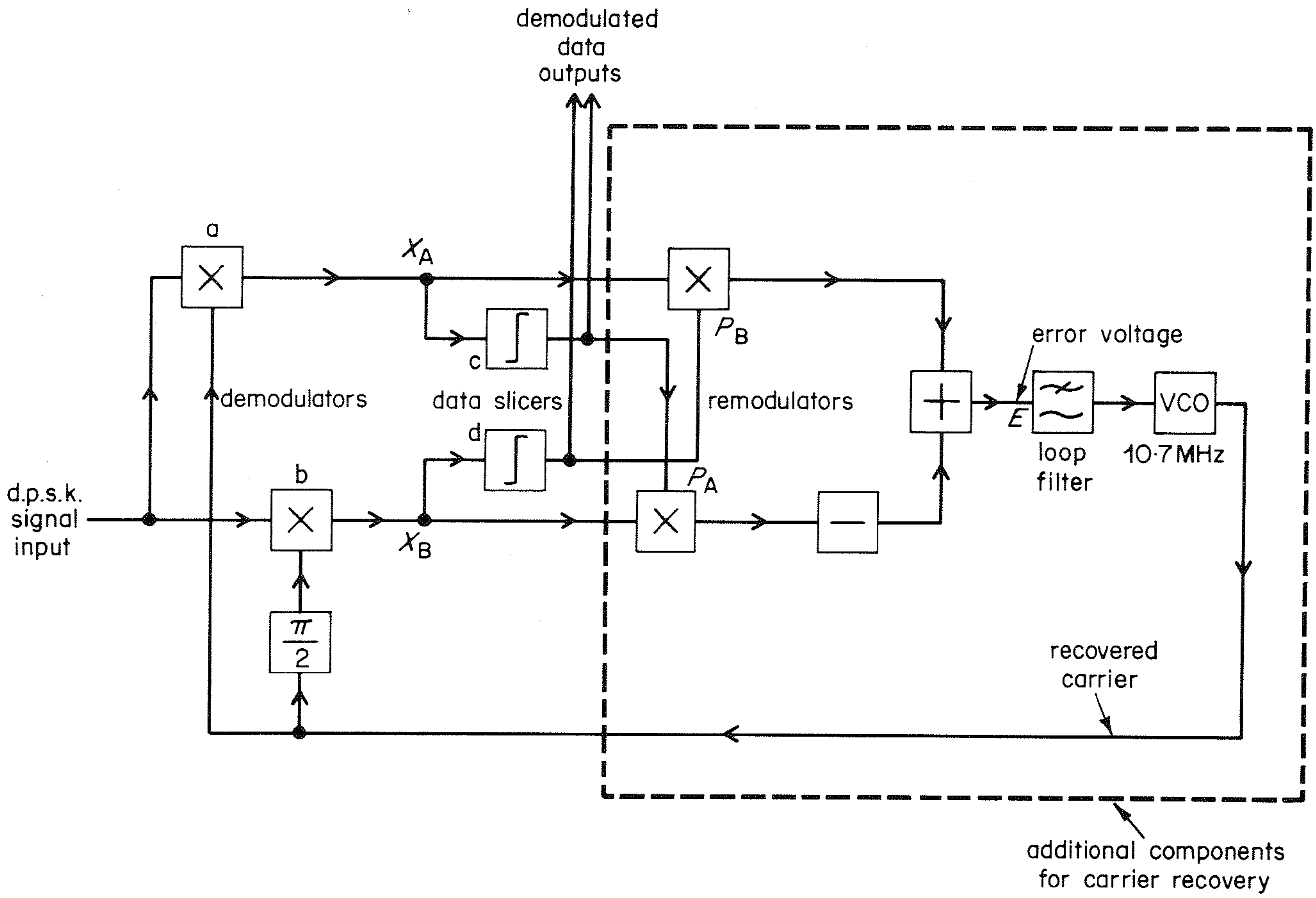

Fig. 4(a) - Carrier recovery by baseband remodulation

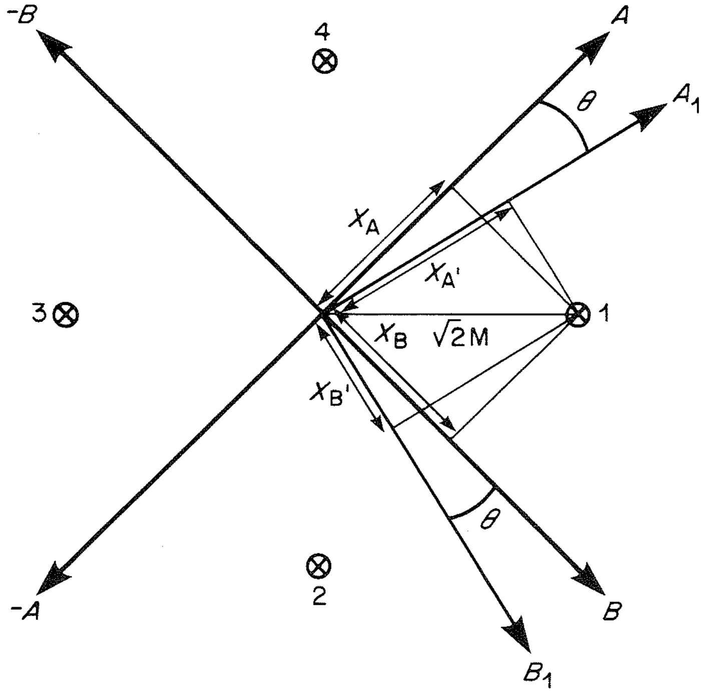

Fig. 4(b) - Vector diagram for carrier recovery by baseband remodulation ition between the states. All of them will function if the proportion of time spent in transition is small compared with the time spent in one of the rest-states but fluctuations are superimposed on the phase-error voltage because of the time spent in transition between the rest-states. Such fluctuations cause modulation-dependent phase-jitter, known as 'pattern jitter' on the recovered carrier. The effect of the 'pattern jitter' is to slightly reduce the noise margin of the d.p.s.k. signal. However, in general, 'thermal noise' jitter has a much greater influence on the noise margin of the signal than 'pattern jitter'.

Regarded in the frequency domain, the signal to which the carrier recovery oscillator is locked is produced by intermodulation between the sidebands of the d.p.s.k. signal. For carrier recovery to be possible a certain number of these sidebands must be included with sufficient amplitude. The input signal bandwidth required to achieve this depends upon the modulation sequence and it can be shown necessary for the bandwidth to be at least 1·5 times the symbol rate for certain worst-case modulation sequences.[1] Raised-cosine spectrum-shaping of the input signal with a bandwidth between extinction points of twice the symbol-rate meets this criterion. If narrower bandwidths are to be used it may be necessary to ensure that certain modulating sequences do not occur for a significant length of time.

4. Comparison of carrier recovery methods

4.1. General

The only performance parameters of the carrier recovery unit that affect the resistance to noise and multipath interference of the complete d.p.s.k. receiver, assuming that the carrier-recovery loop has achieved lock, are:

(i) The static phase-accuracy of the recovered carrier
(ii) Phase jitter of the recovered carrier.

All the methods of carrier recovery described above can, theoretically, be designed to meet the requirements in these respects but they differ in terms of ease of instrumentation, and ruggedness in operation. Features of the three methods are compared in the following sections.

4.2. Sensitivity to instrumental phase drifts

The phase-multiplication method of carrier recovery is an open-loop system. With reference to Fig. 2, any phase drifts in the system between the signal input and the phase comparator will cause a drift in the phase of the recovered carrier. When a prototype model[1] was made it was found that the bandpass filters were particularly prone to temperature drift, necessitating readjustment of the output phase from time to time. Both remodulation methods use closed loops and, provided the two branches of the loops corresponding to the two demodulation axes remain balanced, internal phase drifts will automatically be compensated, and will not result in a phase error of the recovered carrier.

4.3. Dependence upon modulating sequence

The modulation on the d.p.s.k. signal affects the amplitudes and distribution of its sidebands. The amplitude of the component to which the carrier-recovery voltage-controlled oscillator is locked, both in the phase-multiplication method and in either of the two remodulation methods, changes with the modulating sequence since it is produced by intermodulation among these sidebands. The range of this amplitude variation is much greater with the phase-multiplication method than with either of the two remodulation methods because of the fourth-power amplitude law employed in the phase-multiplication method. The loop-gain* of the phase-locked loop is proportional to the amplitude of its input signal. The loop-gain affects the various phase-locked loop performance parameters and consequently the overall performance of the unit is much more sequence-dependent in the case of the phase-multiplication method than for either of the two remodulation methods.

4.4. Sensitivity to level of input signal

The level of input carrier to the phase-locked loop varies as the fourth power of the input signal in the phase-multiplication method, but only as the first power in either of the two remodulation methods. The requirements for signal-amplitude stability are thus easier to meet with the remodulation methods.

It may seem possible to use an amplitude limiter at the input to the phase-locked loop (Fig. 2) to reduce the sensitivity of the phase-multiplication method to both the modulating sequence and the input signal level. However, the input signal of the limiter would not be a single frequency, but would be composed of the four-times carrier frequency component, along with many sidebands. A limiter is likely to cause phase errors and thus result in a degradation of the d.p.s.k. receiver performance.

4.5. The phase-error function

The 'phase-error function' is the error voltage developed by the phase-detection circuitry, expressed as a function of the difference between the phase of the vector A (Fig. 1) and the phase of the output of the voltage-controlled oscillator. Fig. 5 compares this function for the phase-multiplication method with that for the two remodulation methods.

The phase-error function for the phase-multiplication method is $\sin 4\theta$ where θ is the phase error *at carrier frequency*. The angle 4θ occurs since the phase comparison is at four-times carrier frequency. The remodulation methods have an error function which changes discontinuously from $\sin\theta$ to $\cos\theta$ at 90° intervals. This approxi-

---

* Loop-gain is proportional to the error voltage produced by the phase-locked loop phase-comparator, per unit phase-error.

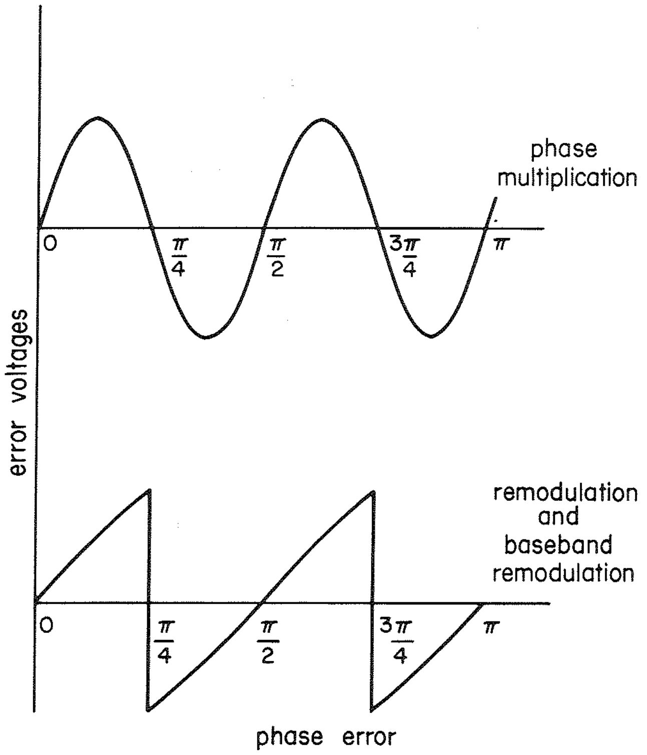

Fig. 5 - Phase-error functions for phase-multiplication and remodulation methods

mates to a linear sawtooth form and results in faster pull-in.[2]

4.6. Simplicity of instrumentation

The phase-multiplication method requires two highly-stable bandpass filters and two wide-bandwidth square-law devices, together with a high-speed frequency-divider. Many of these components, as well as the voltage-controlled oscillator, operate on signals at four-times carrier frequency (42·8 MHz), making the design costly to implement.

Both remodulation methods require fewer components; the baseband method, in particular, requires only a few components over and above those required to demodulate the signal. (The extra components required, for both remodulation methods, are indicated in Figs. 3(a) and 4(a)). The data slicers used to reconstitute the digital signal can also be used for carrier recovery, while the 'remodulators' are merely polarity-inverting switches. Most of the components in the conventional remodulation method (Fig. 4(a)) operate on carrier-frequency signals (10·7 MHz), and one bandpass filter is required. No bandpass filters are required for the baseband remodulation method and most of the components operate at baseband frequencies. This method is therefore the cheapest to implement. It would also be the most suitable for large-scale integration for use in domestic receivers should 4-phase d.p.s.k. broadcasting become a reality in the future.

The baseband remodulation method suffers from one instrumental disadvantage. It is d.c.-coupled throughout, requiring careful attention to d.c. offsets. This makes the method difficult to instrument at high carrier-frequencies or at high bit-rates, when components such as linear multipliers are difficult to realise. Such questions do not apply, however, in the present application.

5. Theoretical and measured performance of prototype units

Since, as discussed in Section 4, the two remodulation methods of carrier recovery are so similar, and the baseband-remodulation method is the easier of the two to implement, further consideration was given only to the phase-multiplication and the baseband-remodulation methods. Accordingly, a prototype unit for each of the two methods was designed* and tested.

The theoretical and measured performances of the two prototype units are given in Table 1. Theoretical performances are given for an unmodulated carrier and for a carrier modulated at symbol rate by progressive $\pi/2$ phase-shifts (giving the maximum and minimum loop gains respectively). The measured performance is given for an unmodulated carrier; a signal with pseudo-random modulation, which is typical of the modulation that will be used in practice, was found to give similar results.

The change in loop-gain of the phase-locked loop, between the unmodulated carrier case and the progressive $\pi/2$ phase-shifts case, was also measured. The range of variation was 6:1 with the phase-multiplication unit, but only 1·6:1 with the remodulation unit. The variation of phase-locked loop parameters with modulating sequence can be calculated using these ratios. The larger the loop-gain ratio, the larger the variation in performance parameters. The values shown in Table 1 for progressive $\pi/2$ phase shifts were calculated from the unmodulated carrier values using the measured ratios. It must be emphasised however, that these ratios are a function of the spectrum shaping of the signal, and of the characteristics of the two bandpass filters used in the phase-multiplication method. If the bandwidths were all infinite, that is if the signal phase-transitions were instantaneous, the ratios would both be 1:1. The gain ratios are increased as bandwidths are narrowed. Raised-cosine spectrum shaping was used on the signal for the tests described here. The reasons for this choice are given in Ref. 1.

The measured variation of phase jitter on the recovered carrier is shown in Fig. 6, as a function of input signal-to-noise ratio. At low signal-to-noise ratios, the jitter was independent of the modulating sequence. As the signal-to-noise ratio was improved, the jitter progressively reduced when using an unmodulated carrier but, when using a modulated carrier, it asymptotes to a value equal to the 'pattern jitter' (see Section 3.5.).

---

* Design details for the baseband-remodulation unit are given in Appendix II.

TABLE 1

Theoretical and measured performances of prototype carrier-recovery units

Parameter	Phase-multiplication unit			Baseband-remodulation unit		
	Calculated		Measured	Calculated		Measured
	Unmodulated carrier	Progressive $\pi/2$ phase-shifts	Unmodulated carrier or pseudo-random modulation	Unmodulated carrier	Progressive $\pi/2$ phase-shifts	Unmodulated carrier or pseudo-random modulation
Mean phase error for 50 KHz error in carrier frequency	0·17°	0·8°	0·2°	0·3°	0·5°	0·4°
Phase jitter at carrier/noise ratio of 13·4 dB[+] (See also Fig. 6)	x°[*]	6x°[*]	1·8°	2·0°	3·2°	2·2°
Pull-in range	±75 KHz	±12·5 KHz	±27 KHz	±120 KHz	±75 KHz	+70 KHz −160 KHz

[*] It is difficult to calculate the phase jitter for the phase-multiplication method but its ratio with and without the modulation was expected to be as shown.

[+] 13·4 dB carrier to noise ratio corresponds to an error rate of 1 in 10^5, in the absence of recovered carrier phase jitter. At this error rate, 2·7° r.m.s. phase jitter results in 0·2 dB impairment in signal to noise performance.

As measured, the phase-multiplication unit gave less jitter and less static phase error than that given by the remodulation unit, but it also gave a smaller pull-in range. The gain and bandwidth of the remodulation unit could have been altered, however, to have less phase error and better jitter performance than the phase-multiplication unit while still giving better pull-in range. Since, moreover, its performance changes so little with modulating sequence, the baseband remodulation unit is considered the best on all counts.

There was little thermal phase-drift in the recovered carrier with the baseband remodulation unit, in contrast to the significant phase-drift that was found with the phase-multiplication unit.[1]

'Cycle skipping'[1] was present on both prototypes at about the same phase jitter value (7° r.m.s.); the phase-locked loop falls momentarily out of lock and then re-locks in a different phase. Hold-in range, like pull-in range, was better with the remodulation unit than with the phase-multiplication unit, but no measurements were made since target specifications were met in each case.

6. Practical aspects of baseband remodulation carrier recovery unit design

Because of the features described above, baseband remodulation is the recommended method of carrier recovery for future d.p.s.k. sound systems of the type considered in this report. The following practical details will be of interest to the designers of such units.

It was found that the switching signals P_A and P_B fed to the remodulators of the remodulation unit (see Fig. 4(a)) needed to be timed relative to the baseband signals X_A and X_B to within about 0·5% of each symbol interval in order to avoid phase errors when demodulating certain modulating sequences. Furthermore, for the case of the baseband remodulation unit, particular attention needed to be paid to d.c. offsets. No degradation due to d.c. offsets was evident when the loop was in lock but, when there was no input signal, the high open-loop gain was found to amplify the offsets sufficiently to cause the voltage-controlled oscillator to drift significantly from its correct frequency and thus make subsequent pull-in difficult. To overcome the problem a circuit could be

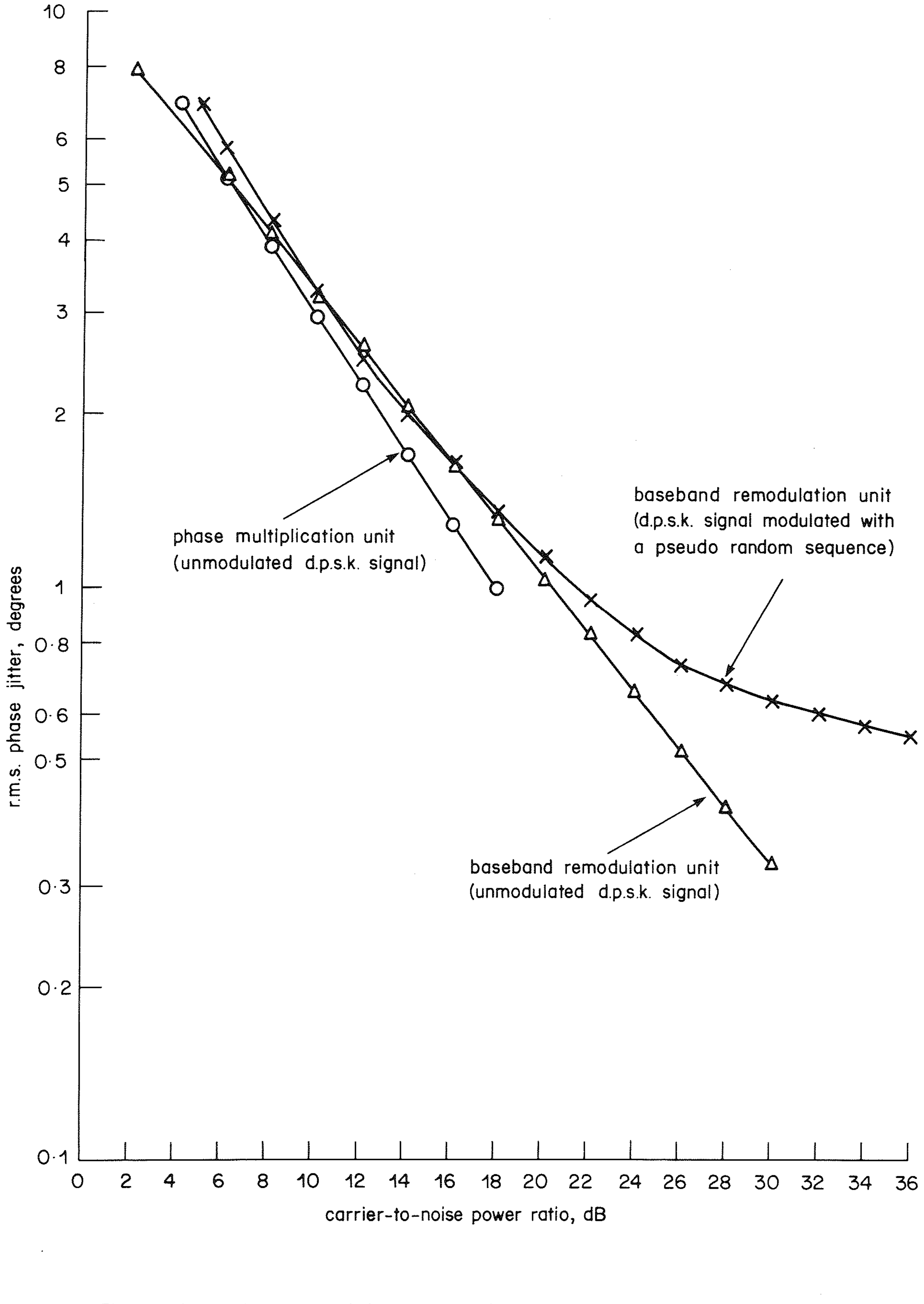

Fig. 6 - Measured variations of phase jitter as a function of input signal-to-noise ratio

incorporated that would restrict the drift to, say, ±70 KHz but this would then equally restrict the pull-in range.

Certain modulating sequences have large-amplitude sidebands near the carrier frequency and, if such a sequence occurs whilst the carrier-recovery unit is coming into synchronism the unit may lock to one of these sidebands rather than to the carrier, causing what is termed a "false lock". Both the remodulation and the phase-multiplication types of carrier-recovery unit are equally susceptible to this but it is likely to be of little consequence in practice because the unit will move into the correct lock on receipt of the great majority of subsequent modulating sequences. During the tests, no false locks could be induced on either unit when using a pseudo-random modulating sequence. If, however, the application of the unit is such that a modulating sequence which might cause a false lock (such, for example, as progressive $\pi/2$ phase-changes) frequently occurs, some form of anti-false-lock circuitry may be necessary. A false lock can be recognised because:

a) the amplitude of the demodulated signal is $\sqrt{2}$ times larger than when in true lock

and b) the phase-error voltage within the phase-locked loop has a strong component at a frequency equal to four times the difference between the true carrier frequency and the voltage-controlled oscillator frequency.

Should such evidence occur, the anti-false-lock circuitry should be arranged to disturb the loop so as to throw it out of lock.

7. Conclusions

The work described in this report has shown that the baseband-remodulation method of carrier recovery, for synchronous demodulation of a d.p.s.k. signal, can provide the required performance more easily than either the phase-multiplication or the carrier-remodulation method. It has also been shown that the phase jitter on the demodulating carrier recovered using the baseband remodulation method reduces the noise margin of the d.p.s.k. system by less than 0·2 dB.

Both remodulation methods are cheaper to instrument than the phase-multiplication method and are also less sensitive to instrumental drifts, modulating sequence and variations of input signal level. The baseband-remodulation method is cheaper to instrument than the remodulation method because most of its circuitry operates at baseband frequencies rather than at the carrier frequency. It is also perhaps the most suitable for large-scale integration. Being d.c.-coupled throughout, however, its design requires careful attention to d.c. offsets, but this does not present great difficulty.

For the above reasons, the baseband-remodulation method of carrier recovery is recommended for the proposed 4-phase d.p.s.k. sound system.

8. References

1. KALLAWAY, M.J. 1976. An experimental 4-phase differential-phase-shift-keying system to transmit two high-quality sound signals. BBC Research Department Report No. 1976/20.

2. WOLSJSZA JR, C.J., WALKER, A.M. *and* WERTH, A.M. 1969. PSK modems for satellite communications. Proc. of the Intelsat/IEE International Conference on Communications, November 1969, pp 127-143.

3. NOZAKA, K., MARATONI, T., OGI, M., *and* SHOJI, T. 1969. Carrier synchronisation techniques of p.s.k. modem for TDMA systems. Proc. of the Intelsat/IEE Conference on Communications, November 1969.

4. LUNDQUIST, L., LOPRIORE, M. *and* GARDNER, F.M. 1974. Transmission of 4-phase shift-keyed time-division multiple access over satellite channels. IEEE Transactions on Communications Vol. Com-22, 9; September 1974.

5. GARDNER, F.M. 1966. Phase-lock techniques, New York, J. Wiley, 1966.

9. APPENDIX I

Equivalence of the conventional remodulation method and the baseband remodulation method of carrier recovery

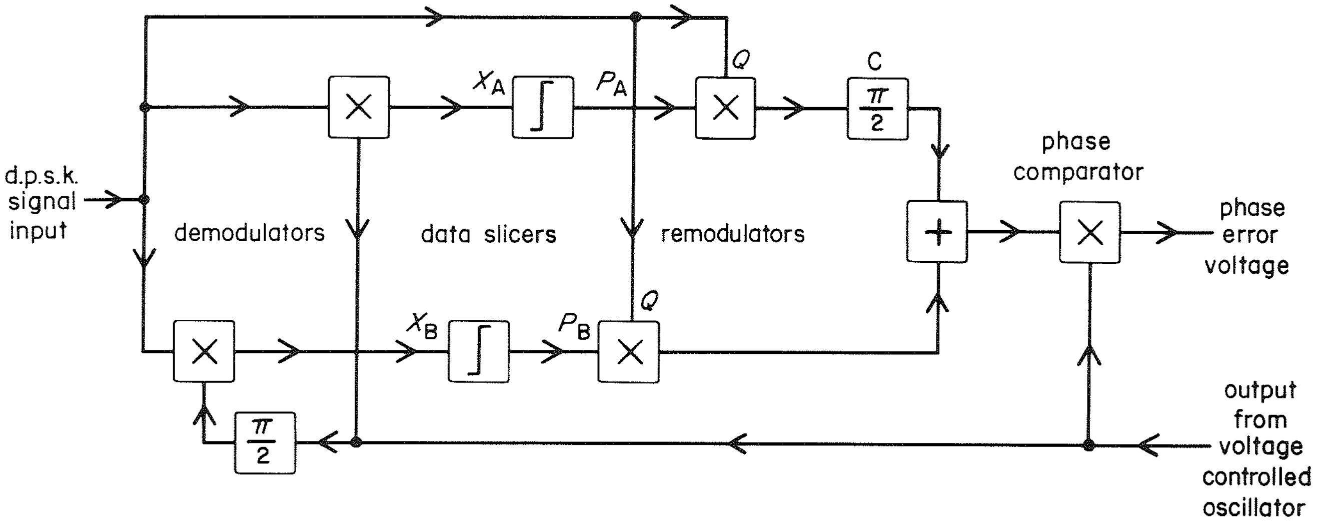

Fig. 7 - Heart of remodulation carrier recovery

The following analysis shows that the conventional remodulation and baseband remodulation methods of carrier recovery are equivalent.

The 'heart' of the convention remodulation method of carrier recovery is shown in Fig. 7. Fig. 8 shows a variant of the arrangement of Fig. 7 which is an exact equivalent. In Fig. 8 the signal Q which feeds the remodulators is regenerated from the demodulated signals X_A and X_B. This re-arrangement of the system has been done to aid the analysis.

Referring to Fig. 8, let the voltage-controlled oscillator output be $e^{j\omega t}$.

After a phase lag of $\pi/2$ this becomes $-je^{j\omega t}$.

Hence the d.p.s.k. signal Q regenerated from X_A and X_B in Fig. 8 is given by $X_A\ e^{j\omega t} - j\ X_B\ e^{j\omega t} = (X_A - jX_B)\ e^{j\omega t}$.

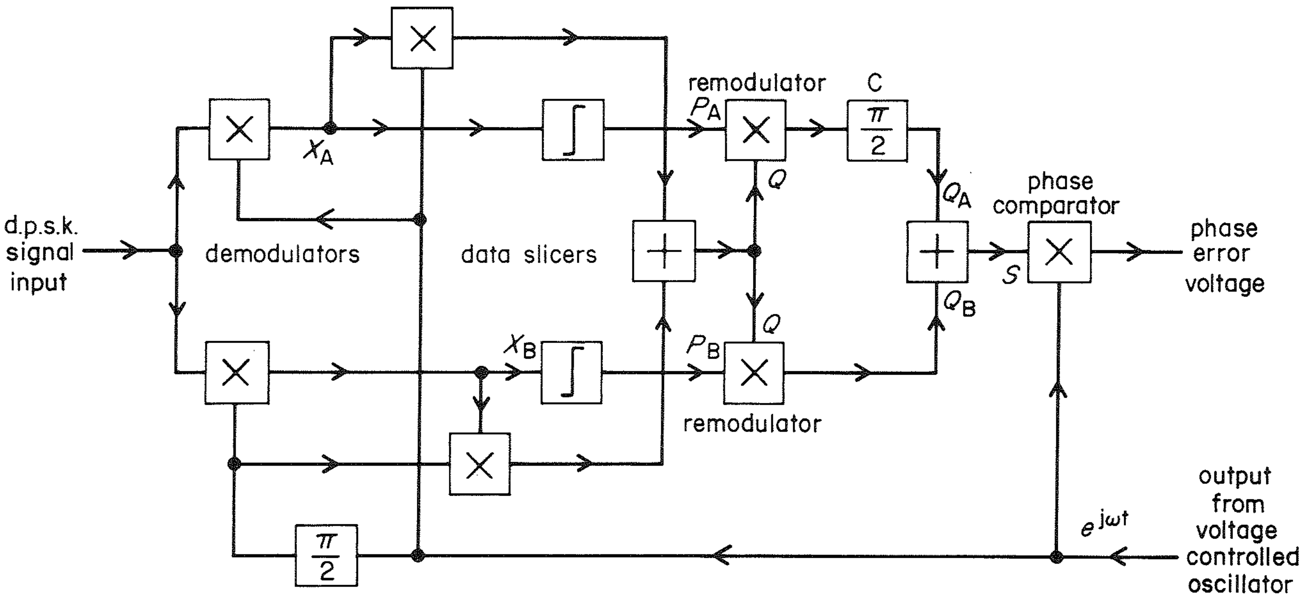

Fig. 8 - Heart of remodulation carrier recovery, equivalent circuit for analysis

This signal is multiplied by P_A and P_B respectively in the two remodulators to give the signals Q_A and Q_B where:

$$Q_A = -jP_A\ (X_A - jX_B)e^{j\omega t}$$

(where the effect of the phase shifter C is included) and $Q_B = P_B\ (X_A - jX_B)\ e^{j\omega t}$.

The signals Q_A and Q_B are then summed to give the signals S given by:

$$S = (P_B X_A - P_A X_B) - j(P_A X_A + P_B X_B)e^{j\omega t}$$

In the phase comparator this signal is multiplied with the voltage-controlled oscillator output $e^{j\omega t}$. The baseband component of the output from the phase comparator will be

E = Baseband component of $\left[\text{Real part } [S] . \quad \text{Real part } [e^{j\omega t}]\right] = \frac{1}{2}\ (P_B X_A - P_A X_B)$.

The error voltage, E, produced by the baseband remodulation unit, shown in Fig. 4(a) is equal to $(P_B X_A - P_A X_B)$. Hence, the baseband remodulation method is an equivalent of the conventional remodulation method of carrier recovery.

10. APPENDIX II

Baseband remodulation carrier recovery unit: Analysis and design

10.1. Analysis

The design procedure for methods other than baseband remodulation is not considered here since it is not envisaged that they will be used in the final version of the d.p.s.k. modem.

The baseband remodulation method of carrier recovery behaves very much like a conventional phase-locked loop (p.l.l.). The operation is outlined in Section 3.4., and described below in greater detail.

Fig. 4(a) is a block diagram of the system and Fig. 4(b) shows the phase relationships of the signals within the system. In Fig. 4(b) the four phase-states of the d.p.s.k. signal are labelled 1 to 4. The reference and quadrature feeds of the demodulating carrier are shown by the vectors A_1 and B_1. The phase error of these signals from their correct phase is θ.

Referring to Fig. 4(b), if the phase of the demodulating carrier was correct ($\theta = 0$), the outputs of the demodulators would be X_A and X_B when the d.p.s.k. signal was in phase-state 1. Let the amplitudes of X_A and X_B = M.

When there is a carrier phase error of θ, the outputs of the demodulators are X_A' and X_B' where:

$$X_A' = \sqrt{2}\mathrm{M}\cos\left(\frac{\pi}{4} - \theta\right)$$

and

$$X'_B = \sqrt{2}\mathrm{M}\cos\left(\frac{\pi}{4} + \theta\right)$$

Also, since X'_A and X'_B are both positive, the outputs P_A and P_B from the data slicers in Fig. 4(a) are both +1.

Hence the error voltage E in Fig. 4(a) is given by

$$E = P_B X'_A - P_A X'_B = \sqrt{2}\mathrm{M}\left[\cos\left(\frac{\pi}{4} - \theta\right) - \cos\left(\frac{\pi}{4} + \theta\right)\right]$$

$$= 2\mathrm{M}\sin\theta$$

Similarly, it can be shown that the error voltage is 2M sin θ, when the d.p.s.k. signal is in phase-states 2, 3 and 4.

Hence, referring to Fig. 4(a), the circuitry from the input demodulators to the production of the error voltage can be considered as a phase comparator.

10.2. Performance of a linearised model

Starting from the description above, the baseband remodulation carrier recovery unit can be represented by the linearised model shown in Fig. 9, for an unmodulated d.p.s.k. signal. Formulae for predicting the performance of this model are quoted below, without proof. The formulae are based on those derived in Ref. 5, where the analysis of phase-locked-loops is done with the aid of Laplace transforms. The following notation is used in the analysis (see Fig. 9).

K_d = phase comparator gain factor in units of volts per radian of phase error

K_o = voltage-controlled oscillator gain constant in units of angular frequency change (radians sec^{-1}) per volt of control voltage.

$F(s)$ = transfer function of the loop filter, where

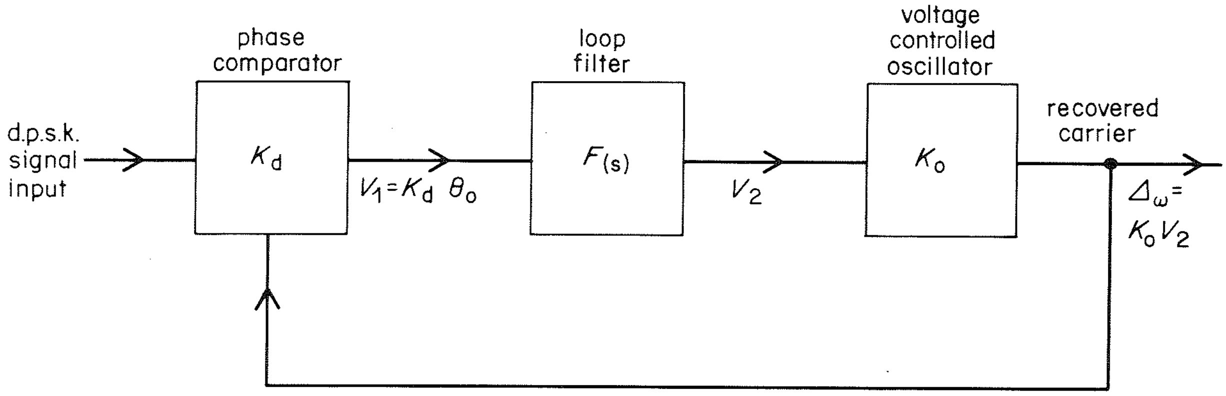

Fig. 9 - Linearised model of carrier recovery by baseband remodulation

s is the complex frequency variable

In the prototype unit the loop filter used had a transfer function of the form

$$F(s) = \frac{G(1 + s\tau_2)}{(1 + s\tau_3)}$$

where

G is the d.c. gain and

τ_2, τ_3 are time constants.

This type of loop filter gives the phase-locked-loop a second-order response characterised by a natural frequency ω_n and damping factor ζ.

In terms of the parameters of the p.l.l., the natural frequency of the loop

$$\omega_n = \left(\frac{K_o K_d G}{\tau_3}\right)^{1/2} \tag{2}$$

and the damping factor of the loop

$$\zeta = \tfrac{1}{2}\omega_n \left(\tau_2 + \frac{1}{K_o K_d G}\right) \simeq \tfrac{1}{2}\omega_n \tau_2 \text{ for a high gain loop} \tag{3}$$

The formulae for the calculation of the various loop performance parameters are quoted below.

(i) Static phase error per unit frequency offset

$$\text{Phase error} = \frac{1}{G K_o K_d} \text{ radians/radians sec}^{-1} \tag{4}$$

(ii) Phase jitter on recovered carrier due to noise

$$\overline{\theta^2}_{no} = \frac{P_n}{P_s}\,\frac{B_N}{B_1} \text{ radians}^2 \tag{5}$$

where

$\overline{\theta^2}_{no}$ = mean square output phase jitter in (radians)^2

B_1 = equivalent noise-bandwidth of the input band-pass-filters preceding demodulation in the d.p.s.k. receiver

P_n = input noise power (in the bandwidth B_1)

P_s = input signal power

B_N = phase-locked-loop single-sided noise-bandwidth which can be calculated using formula (6).

The single-sided noise-bandwidth of the p.l.l. B_N is given below.

$$B_N = \frac{\omega_n}{2}\left(\zeta + \frac{1}{4\zeta}\right) \text{Hz} \tag{6}$$

(B_N is in Hz despite the fact that ω_n is in radians sec^{-1}).

This formula is applicable to high gain loops provided that the input noise-bandwidth is much wider than the loop noise-bandwidth (i.e. $B_1 >> B_N$). Both conditions are satisfied in our application. Formulae (5) and (6) are derived from various assumptions which are explained in Ref. 5 pp 17-22.

(iii) Pull-in range

For a high gain second-order loop

$$\text{pull-in range} \quad f_p = \pm \frac{1}{2\pi}\left[\zeta K_o K_d G\, \omega_n\right]^{1/2} \text{ Hz} \tag{7}$$

In the case of a carrier recovery unit this formula will apply only for the case of an input d.p.s.k. signal which is either unmodulated or randomly modulated. Non-random modulation produces coherent sidebands to which the loop can lock, and in practice the pull-in range is less than that predicted because of the existence of false-locks. It should also be noted that the formula is in any case empirical and very approximate. Further information on the tracking and acquisition performance of phase-locked-loops can be found in Ref. 5 (Chapter 4).

10.3. Design objectives

The following were target design parameters.

(i)	Static phase error	<0·5° for 50 kHz frequency offset	with an unmodulated d.p.s.k. signal input
(ii)	Phase jitter	<2·7° r.m.s. at an input carrier-to-noise ratio of 13·4 dB. (This results in less than 0·2 dB degradation of the noise margin of the modem at a noise level at which the bit-error rate is 1 in 10^5)^1	
(iii)	Pull-in range	>± 50 kHz	
(iv)	Hold-in range	>± 50 kHz	

(v) Loop damping factor $0{\cdot}7 < \zeta < 2{\cdot}0$ for all possible modulating sequences.

10.4. The design of the experimental circuits

10.4.1. Basic circuit parameters

The phase-comparator gain

K_d = 2M for small values of θ. (See Section 10.1.).

For the experimental circuit M = 1 Volt, and hence

K_d = 2 volts/radian

When the carrier is modulated with a continuous series of $\pi/2$ phase shifts this drops to

K_d = (2)/(1·6) = 1·25 volts/radian. This is the minimum value of phase comparator gain. (See Section 5). The voltage-controlled oscillator gain constant K_o was $2{\cdot}95 \times 10^5$ radians sec^{-1}/volt of control voltage.

10.4.2. Static phase accuracy

Equation (4) can be used to calculate the minimum value of G to give the required static phase accuracy given in Section 10.2. The minimum value of G was calculated as being 61. In order to allow a factor of safety, G was made equal to 100 in the experimental circuit.

10.4.3. Phase jitter due to noise

Equation (5) can be used to calculate the maximum noise-bandwidth, B_N, required to keep phase jitter within the specified limit, given in Section 10.2. For the experimental circuit

P_s/P_n = 13·4 dB = 21·9, for an unmodulated d.p.s.k. signal, and

B_1 = $\frac{3}{8}$ x 652 kHz = 245 kHz.*

Hence, to meet the phase jitter specification, B_N must be less than 11·9 kHz.

In order to allow a factor of safety the noise bandwidth of the experimental circuit was

$$B_N = 6{\cdot}5 \text{ kHz}$$

10.4.4. Loop damping factor and natural frequency

The p.l.l. was designed so that the minimum value of the loop damping factor was 0·7. This corresponds to the case when the input d.p.s.k. signal is modulated with a continuous series of $\pi/2$ phase shifts.

From equations (2) and (3) in Section 10.2., it can be seen, that the loop damping factor is proportional to the square root of the d.c. loop gain. In Section 5 it was shown that the d.c. loop gain of the system varies over a 1·6 to 1 range. Consequently the maximum value of the loop damping factor is 0·89 when the d.p.s.k. signal is unmodulated.

The values of the loop damping factor and noise-bandwidth obtained above can be substituted in equation (6) to find the required loop natural frequency. By substitution, the value of ω_n required was calculated as being $\omega_n = 11{\cdot}2 \times 10^3$ radians sec^{-1}.

Also by substituting the values of ω_n and ζ just obtained in equations (2) and (3) the value of τ_2 and τ_3 can be found. The values of τ_2 and τ_3 were calculated as being

$$\tau_2 = 1{\cdot}6 \times 10^{-4} \text{ sec}$$

$$\tau_3 = 0{\cdot}47 \text{ sec}$$

10.4.5. Pull-in range and hold-in range

By substituting the values of ω_n and ζ obtained above into equation (7) the pull-in range of the p.l.l. was calculated as being ±120 kHz (for an unmodulated d.p.s.k. signal). This more than meets the target.

Hold-in range is determined largely by saturation, but will certainly exceed the pull-in range.

10.4.6. The effect on the p.l.l. of modulation on the d.p.s.k. signal

Most of the calculations described above were done for the case of an unmodulated input d.p.s.k. signal. As explained in Sections 4.3. and 5 the open loop gain varies with the modulating sequence. ω_n and ζ are proportional to the square root of loop gain. (See equations (2) and (3)). The performance with modulated signals can be calculated using appropriately modified values of ω_n and ζ. It must also be remembered that P_s/P_n in equation (5) is equal to the input carrier-to-noise ratio only for the unmodulated carrier case, and is inversely proportional to the square of loop-gain for other modulating sequences.

10.5. The experimental loop filter

The circuit of the loop filter used in the prototype is shown in Fig. 10. The relationship between component values and the values of G, τ_2 and τ_3 is also shown.

The values of G, τ_2 and τ_3 derived above were

$$G = 100$$

$$\tau_2 = 1{\cdot}6 \times 10^{-4}$$

$$\tau_3 = 0{\cdot}47 \text{ sec}$$

---

* The input filter[1] in the d.p.s.k. receiver was a 'raised-cosine' bandpass filter. The equivalent noise bandwidth of such a filter is $^3/8$ the bandwidth between extinction points.

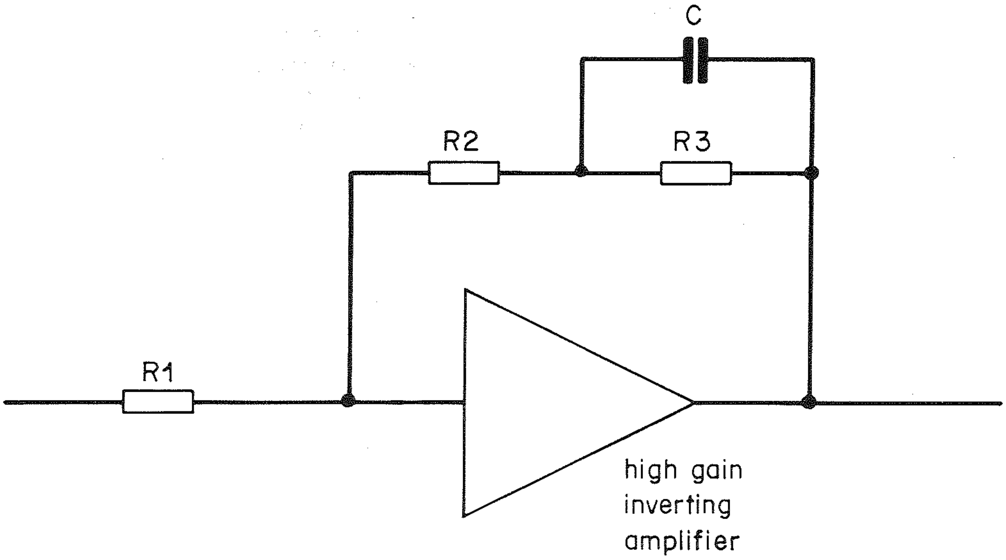

Fig. 10 - Circuit of loop-filter for carrier recovery by baseband remodulation
$G = (R_2 + R_3)/R_1, \tau_2 = R_2C$ and $\tau_3 = R_3C$

Hence, for the filter shown in Fig. 10, the desired response can be achieved by making

$$R_1 = 10\ \text{k}\Omega$$

$$R_2 = 340\ \Omega$$

$$R_3 = 1\ \text{M}\Omega$$

$$C = 0{\cdot}47\ \mu\text{F}$$

10.6. Performance of the experimental circuit

The design targets, the calculated performance, and the measured performance of the experimental circuit are compared in Table 2. The values in the Table apply to the case when the d.p.s.k. signal is unmodulated.

TABLE 2

Parameter	Design target	Calculated value	Measured value
1. Static phase error for 50 kHz frequency offset	< 0·5°	0·3°	0·4°
2. Phase jitter with an input carrier-to-noise ratio of 13·4 dB	< 2·7° r.m.s.	2·0° r.m.s.	2·2° r.m.s.
3. Pull-in range	> ±50 kHz	±120 kHz	+70 kHz −160 kHz
4. Hold-in range	> ±50 kHz	—	Greater than pull-in range

HR/SB

Printed by BBC RESEARCH DEPARTMENT, Kingswood Warren, Tadworth, Surrey, KT20 6NP